A Robust Algorithm for Impulsive Noise Mitigation in OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) systems are susceptible to impulsive noise; in essence, if the impulsive noise is not addressed adequately, system performance loss intensifies by the indelible impairment across OFDM signal bandwidth–thanks to the Fast Fourier Transform operation. This paper capitalizes on the majorization-minimization algorithm in tandem with semidefinite relaxation to avert an overwhelming task engaged in solving a non-convex optimization problem–engendered by the rule that data borne on sub-carriers is subject to a digital modulation format. Simulation results attest the robustness of our proposed receiver: the bit error rate performance level is promisingly close to that of the benchmark receiver–which however uses impulse statistics–in fairly hostile environments.**

***Index Terms*—Majorization minimization algorithm, half-quadratic optimization, semidefinite programming, OFDM, impulsive noise**

I. Introduction

One of the major barriers to communication system design is that the ambient noise in contemporary physical channels is known to be bursty and characterized with a staggeringly large magnitude relative to the additive thermal noise, subjecting communication link quality to unstableness. Orthogonal Frequency Division Multiplexing (OFDM) is remedial to the adversity merely to a certain degree–by spreading noise energy across sub-carriers–through the Fast Fourier Transform (FFT) operation–but is liable to corruption in fairly hostile contexts. As opposed to using the statistics of impulsive noise, a simple approach is to equip the front-end receiver with a memoryless nonlinearity [1], the performance result of which, however, substantially hinges upon a threshold value: for instance, [2] reveals that it is governed by the signal-to-noise ratio (SNR) level and an offset provider.

Though not in a precisely identical context, the impulsive noise in sparse MRI signals, was addressed in [3]. This paper highlights the feasibility of overcoming its negative impact on bit error rate (BER) performance for OFDM systems subject to multipath fading without using the impulse statistics and the level of the SNR ratio as well. By leveraging the majorization-minimization (MM) algorithm [4] for a half-quadratic (H-Q) optimization formulated by a semidefinite relaxation (SDR) problem, our devised framework, through extensive computer simulations, appears to effectively identify the spots corrupted by the impulsive noise and, compellingly counteract the ensuing performance degradation.

Notation: For any vector $\boldsymbol{z} = [z_1, \ldots, z_N]^T \in \mathbb{C}^N$, where $\mathbb{C}$ denotes the set of complex numbers, $\Re(\boldsymbol{z})$ and $\Im(\boldsymbol{z})$ indicate the real and imaginary parts of $\boldsymbol{z}$; further, $(\boldsymbol{z})_k$ denotes the k-th element of $\boldsymbol{z}$, namely z_k; $(\boldsymbol{z})_{i:j} = [z_i, \ldots, z_j]^T$ denotes a subvector of $\boldsymbol{z}$ the elements of which correspond to the range of the subscripted indices. v^* represents the complex conjugate of complex scalar $v \in \mathbb{C}$ and $|v|$ stands for its complex modulus. The transpose of $\boldsymbol{z}$ is denoted by $\boldsymbol{z}^T$ while the conjugate transpose of $\boldsymbol{z}$ is $\boldsymbol{z}^H$. The diagonal matrix $\mathrm{diag}(\boldsymbol{z})$ is constructed by setting its k-th diagonal element to be z_k. The ℓ_1-norm of $\boldsymbol{z}$ is expressed as $\|\boldsymbol{z}\|_1 = \sum_{k=1}^{N} |z_k|$. The trace and inversion of matrix $\boldsymbol{X}$ are denoted by $\mathrm{Tr}(\boldsymbol{X})$ and $\mathrm{Inv}(\boldsymbol{X})$, respectively. $\boldsymbol{A} \succeq \boldsymbol{B}$ indicates matrix $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite. $E(\cdot)$ stands for mathematical expectation.

II. System Model

A codeword is partitioned into segments in accordance with modulation format (the constellation points belong to a modulation set χ[1]. After N consecutive modulated symbols X_m $(0 \leq m \leq N-1)$ are collected, where m is the sub-carrier index for the inverse FFT (IFFT) implementation and N is the IFFT size, the output of IFFT is expressed by $x_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m e^{j\frac{2\pi mk}{N}}$, $(0 \leq k \leq N-1)$. To avoid inter-block interference, the cyclic-prefix (CP) (of which the length L_p is assumed to be longer than the delay spread of the multipath fading channel, i.e., $L_p \geq L$) is inserted at the output of parallel-to-serial (P/S) converter. Let the channel impulse response (CIR) be denoted by $\boldsymbol{h} = [h_0, h_1, \ldots, h_{L-1}]$. With perfect time and frequency control at the receiver, the time-domain received signal sequence (after the CP removal) is subsequently represented by $r_k = h_k \circledast x_k + \eta_k$ $(0 \leq k \leq N-1)$, where $\circledast$ is the N-point circular convolution, and the aggregate non-Gaussian interference sample is represented by

$$\eta_k = i_k + \omega_k \ , \tag{1}$$

where the additive white Gaussian noise (AWGN) is denoted by ω_k. As a convention, the collective impulse noise samples

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[1]For an illustrative purpose, the modulation format is quadrature phase shift keying (QPSK).

correspond to $\boldsymbol{i} = [i_0, i_1, \ldots, i_{N-1}]$, and each of the AWGN noise samples $\boldsymbol{\omega} = [\omega_0, \ldots, \omega_{N-1}]$ has a flat single-sided power spectral density of height N_0, namely $E(|\omega_k|^2) = N_0$.

As opposed to some prevalently known work, it is worth noting that this paper addresses performance loss without assuming the impulsive noise model. One of the popularly used statistical models to construct the memoryless impulsive noise is the Gaussian mixture model. For instance, Bernoulli-Gaussian (B-G) model [5] takes the impulse occurrence $i_k = b_k g_k$ (c.f.,(1)) into account by introducing an inherent Bernoulli random variable $b_k \in \{0, 1\}$–with the probability of impulse occurrence denoted by $P(b_k = 1) = p_b$. Further, the impulsive noise sequence g_k , $k \in \{0, \ldots, N-1\}$ is also an independent and identically distributed (*i.i.d.*) Gaussian random process with mean zero and variance $N_0\Gamma$, where Γ, a strength indicator of the impulsive noise, is the mean power ratio between the impulsive noise g_k and the AWGN noise ω_k, or abbreviated as IGR . As a consequence, the probability density function (PDF) of the noise sample η_k (a two-state Gaussian mixture model) is written as follows:

$$P_{BG}(x) = (1-p_b)\mathcal{CN}(x; 0, N_0) + p_b\mathcal{CN}(x; 0, N_0(1+\Gamma)) \ ,$$

where $\mathcal{CN}(x; \mu_z, \sigma_z^2) = (\pi\sigma_z^2)^{-1}\exp\{-|x-\mu_z|^2/\sigma_z^2\}$ is the complex Gaussian PDF with mean μ_z and variance σ_z^2. To use the statistical knowledge of the B-G model subjects a refined receiver to perfectly estimate the underlying parameters p_b, Γ, and N_0, all of which generally vary with time, raising the concern about complexity to a great degree. Another widely adopted Gaussian mixture model but with infinite states is Middleton Class-A (MC-A) noise model [6]. The PDF of the i.i.d. noise sample $\eta_k = b_k g_k + \omega_k$ is expressed as follows: $P_{MC-A}(x) = \sum_{\ell=0}^{\infty} \alpha_\ell \ \mathcal{CN}(x; 0, \varphi_\ell)$, where $\alpha_\ell = e^{-A}\frac{A^\ell}{\ell!}$ and $\varphi_\ell = N_0(1 + \frac{\ell}{\Lambda A})$. Typically, A is referred to as the *impulsive index* and Λ is the ratio between the AWGN's mean power, and that of the impulsive noise. Noteworthily, b_j is a Poisson variable with mean A and the regarding probability mass function is expressed as $P(b_j = \ell) = \alpha_\ell$ for all non-negative integers ℓ and the PDF of *i.i.d.* samples g_k is $\mathcal{CN}(g_k; 0, N_0/(\Lambda A))$.

Collecting N successive samples $\boldsymbol{r} = [r_0, \ldots, r_{N-1}]$ (preceded by removing the cyclic prefix) yields the following equation in vector form: $\boldsymbol{r} = \boldsymbol{G}\boldsymbol{X} + \boldsymbol{\eta}$, where $\boldsymbol{X} = [X_0, \ldots, X_{N-1}]^T$, $\boldsymbol{\eta} = \boldsymbol{i} + \boldsymbol{w}$ and $\boldsymbol{G} = \boldsymbol{F}^*\boldsymbol{D}_h$, the product of the FFT matrix $\boldsymbol{F}^*$ and a diagonal matrix $\boldsymbol{D}_h$, the m-th diagonal element of which, denoted by $(\boldsymbol{D}_h)_m$, is $\sum_{k=0}^{L-1} h_k e^{-j\frac{2\pi k m}{N}}$. After conducting FFT over the received signal vector $\boldsymbol{r}$ to render $\boldsymbol{R} = \boldsymbol{F}\boldsymbol{r} = \boldsymbol{D}_h\boldsymbol{X} + \boldsymbol{N}$, where the frequency-domain noise sequence is $\boldsymbol{N} = \boldsymbol{F}\boldsymbol{\eta}$, where $(\boldsymbol{N})_m = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\eta_k \ e^{-j\frac{2\pi m k}{N}}$. Under the B-G noise model [5], the PDF of the transformed noise component $(\boldsymbol{N})_m$, $\forall m \in \{0, \ldots, N-1\}$ is written as

$$P_{(\boldsymbol{N})_m}(x) = \sum_{\ell=0}^{N}\binom{N}{\ell}(p_b)^\ell(1-p_b)^{N-\ell}\mathcal{CN}(x; 0, \sigma_\eta^2[\ell]) \ , \quad (2)$$

where $\binom{N}{\ell} = \frac{N!}{(N-\ell)!\ell!}$, and $N!$ is the factorial of an integer N. Invoking the central limit theorem, the equivalent noise strength conditioned on ℓ impulse occurrence is measured by $\sigma_\eta^2[\ell] = N_0\left(1 + \frac{\ell\Gamma}{N}\right)$. Notably, our work aims to recover the OFDM signaling $\boldsymbol{X}$ contingent on assuming the knowledge of $\boldsymbol{G}$. Analogously, the conventional scheme, as dubbed in Section IV for the sake of BER performance comparison, recovers signaling simply by executing $\text{Inv}(\boldsymbol{D}_h)\boldsymbol{R}$, prior to performing demodulation and demapping. Noteworthily, by neglecting the occurrence of impulsive noise, this entailing performance result substantially degrades because the PDF in (2) is far from being the same as that of AWGN assumed as the sole noise source.

III. Iterative Receiver Against Impulsive Noise

Addressing the rarely occurring impulsive noise, this paper adopts the penalty function $\Phi_{\text{C}}(\boldsymbol{\delta}) = \sum_{k=0}^{N-1}\phi_\alpha(|\delta_k|) = \alpha\sum_{k=0}^{N-1}\phi(|\delta_k|)$, where

$$\phi(|z|) = h_L^2 \log\left(\cosh\left(\frac{|z|}{h_L}\right)\right), \quad \forall z \in \mathbb{C} \quad (3)$$

is the log-cosh function [7]–in this work the input argument is revised to be a complex modulus thanks to complex-valued modulated symbols. $\boldsymbol{\delta} = [\delta_0, \delta_1, \ldots, \delta_{N-1}]^T = \boldsymbol{r} - \boldsymbol{G}\boldsymbol{X}$ is the estimation error vector. In consequence, the task of recovering OFDM signaling corrupted by impulsive noise is casted as a non-convex optimization problem:

$$\begin{aligned} &\text{minimize} && \Phi_{\text{C}}(\boldsymbol{X}) \\ &\text{subject to} && (\boldsymbol{X})_k \in \{\pm 1\} + j\{\pm 1\}, {\scriptstyle k \in \{0,1,\ldots,N-1\}}. \end{aligned} \quad (4)$$

To tackle the inefficiency of solving (4) in large-scale problems, the H-Q optimization is enabled by introducing an auxiliary variable $\boldsymbol{v}$ so as to yield a revised objective function:

$$\Phi_{\text{C}}(\boldsymbol{X}) = \inf_{\boldsymbol{v}} \sum_{k=0}^{N-1}\left\{\frac{1}{2}|\delta_k - v_k|^2 + \psi_\alpha(|v_k|)\right\} \ ,$$

where $\psi_\alpha(|v_k|) = \sup_{u_k \in \mathbb{C}}\left\{-\frac{|u_k - v_k|^2}{2} + \phi_\alpha(|u_k|)\right\}$.

Given $\hat{\boldsymbol{v}}(\boldsymbol{X}_t)$ from the output of the proposed MM-SDP algorithm at its latest iteration (where $\boldsymbol{X}_t$ is the OFDM signal estimate at the t-th iteration), $\boldsymbol{X}$ is updated by searching for the optimal solution to the following problem:

$$\begin{aligned} &\underset{\boldsymbol{X}}{\text{minimize}} && \frac{1}{2}(\boldsymbol{y}_t - \boldsymbol{G}\boldsymbol{X})^H(\boldsymbol{y}_t - \boldsymbol{G}\boldsymbol{X}) + \Psi(\boldsymbol{v}(\boldsymbol{X}_t)) \\ &\text{subject to} && (\boldsymbol{X})_k \in \{\pm 1\} + j\{\pm 1\}, {\scriptstyle k \in \{0,1,\ldots,N-1\}}, \end{aligned} \quad (5)$$

where $\Psi(\boldsymbol{v}) = \sum_{k=0}^{N-1}\psi_\alpha(|v_k|)$ and $\boldsymbol{y}_t = \boldsymbol{r} - \hat{\boldsymbol{v}}(\boldsymbol{X}_t)$. Noteworthily, to address the hurdle facing the non-convex constraint set, a semidefinite relaxation [8] is employed to facilitate the convex optimization problem formulation:

$$\begin{aligned} &\underset{\boldsymbol{S}}{\text{minimize}} && \text{Tr}(\mathcal{L}_t\boldsymbol{S}) \\ &\text{subject to} && \text{diag}(\boldsymbol{S}) = \boldsymbol{1}_{2N+1} \\ &&& \boldsymbol{S} \succeq \boldsymbol{0}_{(2N+1)\times(2N+1)} \ , \end{aligned} \quad (6)$$

where $\boldsymbol{S} = \boldsymbol{s}\boldsymbol{s}^T$ and $\boldsymbol{1}_{2N+1}$ is a tall vector with all of its $(2N+1)$ elements equal to one, and

$$\mathcal{L}_t = \begin{bmatrix} \bar{\boldsymbol{G}}^T\bar{\boldsymbol{G}} & -\bar{\boldsymbol{G}}^T\bar{\boldsymbol{y}}_t \\ -\bar{\boldsymbol{y}}_t^T\bar{\boldsymbol{G}} & \bar{\boldsymbol{y}}_t^T\bar{\boldsymbol{y}}_t \end{bmatrix}, \quad \boldsymbol{s} = \begin{bmatrix} \bar{\boldsymbol{X}} \\ 1 \end{bmatrix}, \text{ and} \quad (7)$$

$$\bar{\boldsymbol{y}}_t = \begin{bmatrix} \Re(\boldsymbol{y}_t) \\ \Im(\boldsymbol{y}_t) \end{bmatrix}; \bar{\boldsymbol{X}} = \begin{bmatrix} \Re(\boldsymbol{X}) \\ \Im(\boldsymbol{X}) \end{bmatrix}; \bar{\boldsymbol{G}} = \begin{bmatrix} \Re(\boldsymbol{G}) & -\Im(\boldsymbol{G}) \\ \Im(\boldsymbol{G}) & \Re(\boldsymbol{G}) \end{bmatrix}.$$

Following the lead of [8], the estimate $\bar{\boldsymbol{X}}_{t+1}$ is rendered, given the solution $\boldsymbol{S}_{t+1}$ to the SDR (6) at iteration $t+1$.

By solving the following unconstrained H-Q optimization problem,

$$\hat{\boldsymbol{v}}(\boldsymbol{X}_{t+1}) = \arg\min_{\boldsymbol{v}} \left\{ (\boldsymbol{\delta}_{t+1} - \boldsymbol{v})^H (\boldsymbol{\delta}_{t+1} - \boldsymbol{v})/2 + \Psi(\boldsymbol{v}) \right\}$$

the update of $(\boldsymbol{v})_k$, $(\forall k \in \{0, 1, \ldots, N-1\})$, can be therefore expressed as $(\hat{\boldsymbol{v}}(\boldsymbol{X}_{t+1}))_k = (1 - h_L \tanh(|(\boldsymbol{\delta}_{t+1})_k|/h_L)/|(\boldsymbol{\delta}_{t+1})_k|)\,(\boldsymbol{\delta}_{t+1})_k$. Prior to activating our proposed algorithm, initialization $\boldsymbol{X}_0 = \boldsymbol{0}_{N\times 1}$ is performed. Moreover, at each iteration, compute and update stopping criteria $\mathrm{DF} = |J_t - J_{t+1}|/J_t$ and $\mathrm{DX} = ||\boldsymbol{X}_{t+1} - \boldsymbol{X}_t||_1/||\boldsymbol{X}_{t+1}||_1$, where $J_t = \Phi_C(\boldsymbol{X}_t)$, and continue iterating until $t > N_B$, a pre-determined integer number, or both conditions, i.e., $\mathrm{DF} \le \lambda_1$ and $\mathrm{DX} \le \lambda_2$ are met, where λ_1 and λ_2 are tolerances. At the last iteration, the OFDM symbol is demodulated, and successively demapped to render an estimated codeword.

IV. Simulation Results

The codeword is 512-bit long and the CIR is a ten-path Rayleigh fading model, the channel gains of which are independently generated, on a per-OFDM symbol basis, with unity variance, and are assumed to be known to the receiver. The α value of the penalty function is set to one, and h_L value (c.f., (3)) to 0.35. The stopping criteria of the proposed method, dubbed MM-SDP algorithm, are $N_B = 20$, and tolerances $\lambda_1 = \lambda_2 = 0.01$. Fig. 1 shows the BER curves in the scenarios of impulsive noise generated according to the B-G ($P_b = 0.02$ & 0.2; IGR = 100) and MC-A ($A = 0.005; \Lambda = 0.5$) models, and which model is chosen is however unknown to our method. Neglecting the impulse occurrence, the conventional scheme (see those dashed lines) fails to recover OFDM signals regardless of the degree to which the impulsive noise is adverse to our underlying system. At $P_b = 0.02$ the BER curve derived from our MM-SDP mechanism (see the solid line marked by "+") nearly overlaps the curve of the benchmark receiver (see the dash-dot line). In the hostile environment (i.e., $P_b = 0.2$), the MM-SDP (see the solid line marked by "$\circ$") and the benchmark receiver perform similarly–especially at the regime of high SNR (i.e., $E(|X_m|^2)/N_0$) levels, reinforcing the notions that our proposed scheme is robust against the unknown and detrimental impulsive noise. Noteworthily, the gap between the BER curve induced by the benchmark receiver and that of the clipping operator [1], [2], [9] (see the dashed line marked by "$\circ$") becomes fairly evident at high SNR values: at a 10^{-4} BER level roughly 5 dB SNR deficit is observed. Similarly, our proposed scheme (see the solid line marked by "$*$") is on par with its benchmark peer (see the dash-dot line) in the MC-A model. Furthermore, compared with the results in the B-G model at $P_b = 0.2$, the performance loss by using the clipping operator to our MM-SDP and benchmark receivers lessens to a certain degree in this mild scenario.

V. Conclusion

Facing impulsive noise without assuming its statistics, our proposed approach, a majorization minimization algorithm in tandem with semidefinite relaxation, solves a half-quadratic optimization problem–designed for a penalty function aimed at curbing the impulsive noise–and simultaneously averts an overwhelming task of seeking an optimum of a non-convex optimization problem. Simulation results show that our method is on par with the benchmark peer, which uses the statistics of impulsive noise, and outperforms the clipping-featured counterpart, the threshold of which inevitably engages with not only refined estimation for the signal-to-noise ratio level and underlying channel gains as well, but also–for the sake of robustness–a sweep over a reference range, imitating that computationally demanding off-line task is requisite.

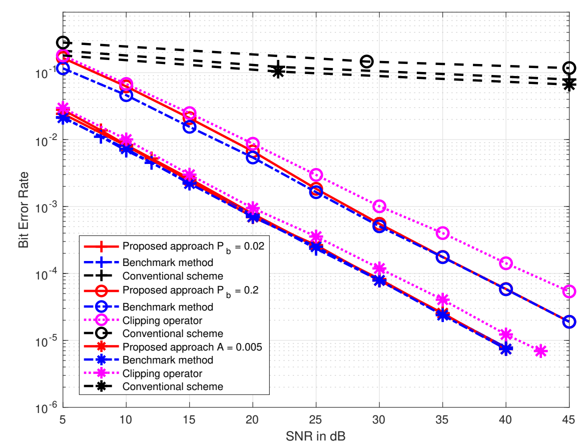

Fig. 1. Comparison of BERs for various approaches in impulsive noise.

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