A Low-Complexity Iterative Detection algorithm for Uplink Massive MIMO Systems

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***Abstract*—Linear minimum mean square error (MMSE) detector achieves nearly optimal performance for massive multiple-input multiple-output (MIMO) systems, but its complexity is significantly high because of the matrix inversion operations involved. Therefore, numerous iterative detectors along with their improved variants such as Gauss-Seidel (GS) have been introduced to tackle computational burden. In this paper, we propose an extrapolated GS (EGS)-based detector to enhance the detection performance. The proposed EGS iteratively refines the initial solution to realize near-MMSE performance without matrix inversion. It improves the convergence rate of the conventional GS detector by incorporating the extrapolation technique. It is demonstrated that the proposed algorithm outperforms the conventional GS and approaches the performance of MMSE algorithm with only a small number of iterations.**

***Index Terms*—Massive multiple-input multiple-output (MIMO), minimum mean square error (MMSE), extrapolation Gauss–Seidel (EGS) method, iterative detector.**

I. Introduction

Massive multiple-input multiple-output (MIMO) has emerged as a crucial technology for advanced wireless communications [1], offering substantial enhancements in link reliability, spectral efficiency, and data rates when compared to small-scale MIMO systems. However, increase in the dimension of massive MIMO and the subsequent rise in the number of served users introduce challenges in terms of increased complexity within the signal processing pipeline. One of the key signal processing challenges is to minimize the computational cost associated with high-dimensional signal separation at the receiver. The complexity of detection process grows exponentially with the number of antennas. To achieve a good trade-off between complexity and performance, one of the widely employed linear detectors is minimum mean square error (MMSE) [2]. It leverages the knowledge of the channel and the received signal vector to estimate the transmitted information with improved accuracy. Linear MMSE achieves a good tradeoff between performance and complexity, but involves matrix inversion. As the number of antennas increases, inversion operation becomes challenging and results in greatly increased complexity.

Numerous works have been dedicated to study the iterative signal detection techniques in order to reduce the computational burden in massive MIMO systems. In [3], [4], the approximate approaches that reduce the complexity of linear detection methods by approximating the large matrix inverse have been proposed. In general, the complexity of these schemes is substantially higher than the iterative algorithms that obtain estimates of the transmitted information symbols without ever computing the inverse of matrix. Thus, there has been significant research interest in exploring different iterative detection methods. To achieve a reasonable detection accuracy, Gauss-Seidel (GS) method has been applied for uplink detection [5] and further modified in [6] to improve the convergence performance. In [7], the accelerated Richardson method-based detector has been developed to address the challenge of high complexity while maintaining low error rates. Furthermore, several other iterative methods such as modified weighted two stage iterative detector [8], preconditioned gradient descent method [9] and preconditioned descent search [10] have been proposed to reduce the complexity of the signal detection.

This work studies an alternative approach based on the extrapolated GS (EGS) method to approximate MMSE detection, which reduces an order of magnitude computational complexity while maintaining high detection accuracy for various massive MIMO system configurations. The EGS method combines the current and previous iteration values with an extrapolation parameter to generate updated estimates. By leveraging information from the previous iterations, the EGS-based detection achieves faster convergence compared to the conventional GS method. Furthermore, to expedite the convergence speed of the proposed algorithm, an antenna based initial solution is utilized. Numerical results and analysis are also provided by illustrating that the proposed algorithm outperforms the conventional GS detector while reducing a significant computational complexity as compared to MMSE.

II. System Model and Signal Detection

We consider an uplink communication system with a massive MIMO configuration, where each user equipped with a single antenna and independently transmit signals to a base station (BS) with N_r antennas. Let the transmitted signal vector is denoted by $\mathbf{s} = [s_1, s_2, ..., s_{N_t}]^T$, where N_t is the total number of transmit antennas, one for each user. Then, the received signal vector at the BS can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ represents the channel matrix and $\mathbf{n} \in \mathbb{C}^{N_r}$ is the additive white Gaussian noise (AWGN) vector with mean zero and variance σ^2 for each element.

Massive MIMO receiver is likely to use a signal detector to correctly estimate the original transmitted signal, which has been corrupted by the undesired copies of the signals coming from different directions. One well-known linear detector, known as MMSE, aims at minimizing the mean square error between the accurate transmitted bit streams and the estimated ones. The estimates of the transmitted signal vector using MMSE is obtained as

$$\hat{\mathbf{s}} = \left(\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I}_{N_t}\right)^{-1}\mathbf{H}^H\mathbf{y} = \mathbf{W}^{-1}\hat{\mathbf{y}}. \tag{2}$$

For simplicity, we describe the matched filter vector and regularized Gram matrix as $\hat{\mathbf{y}} = \mathbf{H}^H\mathbf{y}$ and $\mathbf{W} = \mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I}_{N_t}$, respectively.

III. Proposed detector

Although MMSE detection can achieve a favorable performance in massive MIMO systems, a large number of users and antennas at the BS increase the computational burden of the detection process by orders of magnitude. Thus, we design an iterative detector that achieves the MMSE performance with reduced complexity. The properties of massive MIMO, namely the dominance of channel hardening and the Hermitian positive definite (HPD) nature of the MMSE filtering matrix, can be exploited to develop iterative approaches for detection with significantly lower complexity [11]. Consequently, various iterative detection algorithms are being developed or improved to achieve near-optimal error-rate performance.

Since the matrix $\mathbf{W}$ is HPD for massive MIMO systems, we can decompose it as

$$\mathbf{W} = \mathbf{D} + \mathbf{L} + \mathbf{U}, \tag{3}$$

where $\mathbf{U}$ and $\mathbf{L}$ denote the strict upper and strict lower parts of matrix $\mathbf{W}$, respectively, and $\mathbf{D}$ denotes its diagonal part. If we denote i as an iteration number, the GS method can estimate the transmitted data symbols with the following update equation:

$$\mathbf{s}^{(i+1)} = \mathbf{s}^{(i)} + (\mathbf{D}+\mathbf{L})^{-1}\hat{\mathbf{y}} - (\mathbf{D}+\mathbf{L})^{-1}\mathbf{W}\mathbf{s}^{(i)}, \tag{4}$$

where $\mathbf{s}^{(0)}$ corresponds to the initial solution which will be discussed in sequel. Now let $\mathbf{W}$ be a nonsingular matrix and split $\mathbf{W} = \mathbf{M} - \mathbf{N}$ with a nonsingular matrix $\mathbf{M}$. Then, a basic iterative method to solve $\mathbf{W}\mathbf{x} = \hat{\mathbf{y}}$ is designed as

$$\mathbf{s}^{(i+1)} = \mathbf{M}^{-1}\left(\mathbf{N}\mathbf{s}^{(i)} + \hat{\mathbf{y}}\right), \tag{5}$$

where $\mathbf{M}^{-1}\mathbf{N}$ is an iteration matrix.

We can associate the extrapolation technique with the GS iteration (4) and an iterative form of (5). As in [12], the extrapolation of GS is given by

$$\mathbf{s}^{(i+1)} = \left((1-w)\mathbf{I} + w\mathbf{M}^{-1}\mathbf{N}\right)\mathbf{s}^{(i)} + w\mathbf{M}^{-1}\hat{\mathbf{y}}, \tag{6}$$

where w is the extrapolation parameter. By substituting the iteration matrix and some additional manipulations, the EGS detector can be written as

$$\mathbf{s}^{(i+1)} = (\mathbf{D}-\mathbf{L})^{-1}\left((w\mathbf{U} + (w-1)\mathbf{D})\mathbf{s}^{(i)} + w\hat{\mathbf{y}}\right). \tag{7}$$

Note that the EGS method improves the convergence rate of the conventional GS by incorporating an extrapolation step. It has been discussed in [7] that the MMSE filtering matrix $\mathbf{W}$ can be set to the following diagonal matrix:

$$\mathbf{W}_{n,m} = \begin{cases} N_r + N_t = \alpha, & n = m, \\ 0, & n \neq m. \end{cases} \tag{8}$$

The above condition indicates that each diagonal component of the matrix $\mathbf{W}$ is approximated by the same constant α. Then, the proposed initial solution is given as

$$\hat{\mathbf{s}}^{(0)} = \frac{1}{N_r + N_t}\mathbf{H}^H\mathbf{y}. \tag{9}$$

The above initialization (9) leads to achieve a desired level of detection performance with a less number of iterations, as compared to the conventional GS detector.

IV. Performance Evaluation

This section presents simulation results aimed at demonstrating the enhancements achieved by the proposed algorithm. We compare the proposed algorithm with GS-based detection scheme. The baseline method used for evaluating the symbol error rate (SER) performance is the linear MMSE with exact matrix inversion. We consider 64-quadrature amplitude modulation (64-QAM) for all simulations with different massive MIMO configurations. Channel matrices are generated using a flat Rayleigh fading channel model and extrapolation parameter is chosen as $w = 1.08$.

Fig. 1 presents a comparison involving the proposed algorithm, the classical GS method, and MMSE in a scenario featuring $N_r \times N_t = 128 \times 16$ antenna elements. Even if both the GS and proposed schemes showcase enhanced performance with increasing iterations, note that the conventional GS scheme's performance deteriorates compared to the proposed one. This performance gap becomes more pronounced at higher SNR values. The proposed EGS approach outperforms the GS method by around 0.45dB at a SER of 10^{-3} for $i = 2$, demonstrating the improved detection accuracy of the proposed algorithm within the specified system settings. Furthermore, it attains a performance comparable to that of the linear MMSE technique at $i = 3$.

We proceed by examining the SER performance of the proposed EGS and GS techniques within a system characterized by a reduced BS-to-user antenna ratio, as compared to the configuration depicted in Fig. 1. As illustrated in Fig. 2, the performance of the proposed algorithm is very close to that of MMSE exact matrix inversion for just $i = 5$. We further observe that in contrast to the findings in Fig. 1, the performance disparity between the suggested EGS approach and the classical GS method is even more pronounced at high SNR values. It is attributed to the fact that the conventional GS requires a higher BS-to-user antenna ratio to achieve the near-MMSE performance. In other words, the proposed EGS detector converges faster than the conventional GS detection algorithm.

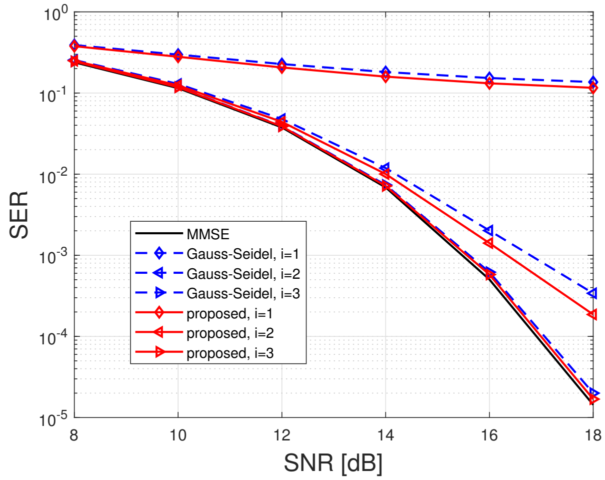

Fig. 1. Comparison of SER of the proposed EGS detector: $N_r = 128$ and $N_t = 16$ (64-QAM).

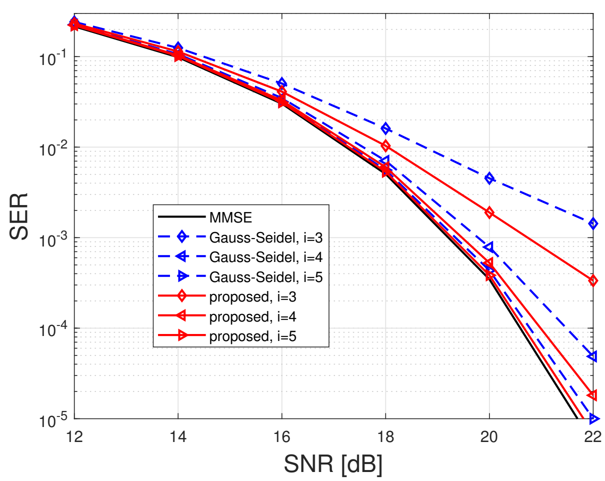

Fig. 2. Comparison of SER of the proposed EGS detector with GS and MMSE: $N_r = 128$ and $N_t = 32$ (64-QAM).

In summary, the proposed algorithm demonstrates significant performance enhancements in terms of error rate compared to conventional GS method and approaches the linear MMSE only with a fewer number of iterations in all scenarios. Moreover, the computational complexity of the proposed detector is lower since it estimates the transmitted signal by iteratively solving the linear equation and only involves matrix-vector multiplications. Its complexity order is $\mathcal{O}(N_t^2)$, which is one order less than MMSE.

V. Conclusion

This paper presented a new signal detection algorithm for massive MIMO systems that builds upon the EGS method. The proposed detector only involves matrix-vector multiplications and reduced the complexity of linear MMSE significantly. Simulation results demonstrate that the proposed EGS outperforms the conventional GS detector in terms of symbol error rate while ensuring relatively faster convergence for small BS-to-user antenna ratios. Moreover, the results of various antenna scenarios illustrate that the proposed algorithm exhibits excellent scalability. This scalability suggests its adaptability to a wide array of antenna setups by simply adjusting the iteration count. Thus, the proposed algorithm stands as a low-complexity and scalable solution for efficient massive MIMO signal detection.

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