

Sparse Channel Feedback for Energy-Efficient mmWave Massive MIMO Systems

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Abstract—mmWave massive multi-input multi-output (MIMO) systems with large-scale antenna array have potential to increase the spectral efficiency by orders of magnitude. One well-known drawback of this systems is that the power consumption of base station (BS) is considerable. To improve the energy efficiency and throughput, it is crucial that the BS acquires accurate downlink channel state information (CSI). However, this task is difficult since the CSI feedback overhead scales linearly with the number of antennas. In this paper, we propose an approach to maximize the energy efficiency of mmWave massive MIMO systems using a sparse channel feedback. Key idea of the proposed scheme is to choose a small number paths among the whole propagation paths in the angular domain channel and then exploit the channel information of chosen paths in the data precoding. From the performance analysis and the numerical results, we demonstrate that the proposed scheme achieves considerable energy efficiency improvement and feedback overhead reduction over the conventional CSI feedback-based schemes.

I. INTRODUCTION

mmWave massive multi-input multi-output (MIMO) systems with large-scale transmit antenna array have received a great deal of attention as a key technology for 5G and 6G wireless systems [1]. However, due to the large number of antennas elements and the radio-frequency (RF) chains such as DA converter and mixer connected to them, energy consumption in mmWave massive MIMO systems is considerable.

In order to achieve the energy-efficient data transmission for the mmWave massive MIMO systems, an acquisition of accurate downlink channel state information (CSI) at the base station (BS) is crucial. Owing to the channel reciprocity, downlink CSI can be easily acquired in time-division-duplexing (TDD) systems. In practice, however, this mechanism will be problematic due to the calibration and synchronization error in the uplink and downlink RF chains [2]. Also, the acquired CSI might be outdated due to the duplexing delay in the TDD systems so that there will be a considerable performance degradation in many fading scenarios [3]. For these reasons, the downlink pilot transmission and channel feedback mechanism is needed for both frequency-division-duplexing (FDD) and TDD systems to guarantee the accurate CSI acquisition. One main concern in the channel feedback is that the feedback overhead scales linearly with the number of antennas. This issue is even more pronounced in the massive

MIMO systems due to the large number of transmit antennas and RF circuitry connected to them [4]–[7].

In this paper, we propose a sparse channel feedback and precoding scheme to minimize the feedback overhead and improve the energy efficiency of mmWave massive MIMO systems. Key ingredient of the proposed scheme is to choose a small number of paths among whole propagation paths in the angular domain channel and then exploit the channel information of chosen paths in the precoding at the BS. The dominating paths are picked among all possible paths to maximize the energy efficiency. From the numerical results, we demonstrate that the proposed scheme achieves a significant energy efficiency improvement over the conventional schemes relying on the CSI feedback.

II. ENERGY EFFICIENCY OF MMWAVE MASSIVE MIMO SYSTEMS

We consider the mmWave massive MIMO systems where a BS with N antennas serve K mobiles with a single antenna. Also, we use the multi-path channel model where the downlink channel vector $\mathbf{h}_k \in \mathbb{C}^N$ of the k -th mobile is expressed as [8]

$$\mathbf{h}_k = \sqrt{\beta_k} \sum_{i=1}^P g_{k,i} \mathbf{a}(\theta_{k,i}) = \sqrt{\beta_k} \mathbf{A}_k \mathbf{g}_k, \quad (1)$$

where P is the number of paths, β_k is the large-scale fading coefficient, $\theta_{k,i}$ is the AoD, and $g_{k,i} \sim \mathcal{CN}(0,1)$ is the path gain of the i -th path, respectively. Also, $\mathbf{a}(\theta_{k,i}) \in \mathbb{C}^N$ is the steering vector, $\mathbf{A}_k = [\mathbf{a}(\theta_{k,1}) \cdots \mathbf{a}(\theta_{k,P})] \in \mathbb{C}^{N \times P}$, and $\mathbf{g}_k = [g_{k,1} \cdots g_{k,P}]^T \in \mathbb{C}^P$ is the path gain information (PGI) vector.

The transmit signal of BS is $\mathbf{x} = \sum_{k=1}^K \mathbf{w}_k s_k \in \mathbb{C}^N$ where $\mathbf{w}_k \in \mathbb{C}^N$ is the precoding vector and s_k is the data symbol for the k -th mobile. Then the power consumption of BS is

$$P_{\text{BS}} = \frac{1}{\eta} \mathbb{E}[\|\mathbf{x}\|^2] + NP_{\text{fix}} = \frac{1}{\eta} \sum_{k=1}^K \mathbb{E}[\|\mathbf{w}_k\|^2] + NP_{\text{fix}}, \quad (2)$$

where $\eta \in [0, 1]$ is the amplifier efficiency and P_{fix} is the power consumption of circuit components (i.e., converters, mixers, and filters). Under this setting, the ergodic achievable rate R_k of the k -th mobile is given by

$$R_k = \log_2 \left(1 + \frac{\mathbb{E}[\|\mathbf{h}_k^H \mathbf{w}_k\|^2]}{\sum_{j \neq k} \mathbb{E}[\|\mathbf{h}_k^H \mathbf{w}_j\|^2] + \sigma_n^2} \right), \quad (3)$$

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where σ_n^2 is the noise variance. Then the energy efficiency E_e is defined as the ratio between the sum rate and the BS power consumption:

$$E_e = \frac{\sum_{k=1}^K R_k}{P_{\text{BS}}}. \quad (4)$$

III. SPARSE CHANNEL FEEDBACK AND ENERGY EFFICIENCY MAXIMIZATION VIA DOMINATING PATH SELECTION

A. Sparse Channel Feedback Via Dominating Path Selection

The proposed sparse channel feedback scheme is divided into two major steps: 1) the BS converts the downlink CSI vector $\mathbf{h}_k \in \mathbb{C}^N$ to the PGI vector $\mathbf{g}_k \in \mathbb{C}^P$ and then 2) the BS converts the PGI vector $\mathbf{g}_k \in \mathbb{C}^P$ to the dominating PGI vector $\mathbf{g}_{\Lambda_k} = [g_{k,i}, \forall i \in \Lambda_k] \in \mathbb{C}^L$ consisting of path gains of L dominating paths. Here Λ_k is the dominating path index set for the k -th mobile (see Fig. 1).

After the selection of dominating paths, a mobile estimates and then feeds back \mathbf{g}_{Λ_k} to the BS. While it looks simple but the dominating PGI acquisition is a bit complicated since the mobile has to decompose all the paths and then estimates their path gains. For this task, we employ a precoded pilot signal with pilot precoding matrix $\mathbf{W}_{p,k}$ designed to convert \mathbf{h}_k to \mathbf{g}_{Λ_k} (i.e., $\mathbf{g}_{\Lambda_k} = \mathbf{W}_{p,k} \mathbf{h}_k$)¹. Using this strategy, the mobile can directly receive the dominating PGI from the precoded pilot signal.

After obtaining the dominating PGI, a mobile feeds back the dominating PGI direction $\hat{\mathbf{g}}_{\Lambda_k} = \mathbf{g}_{\Lambda_k} / \|\mathbf{g}_{\Lambda_k}\|$. To be specific, the k -th mobile chooses a codeword $\mathbf{c}_{\hat{i}_k}$ from a random vector quantization (RVQ) codebook \mathcal{C} and then feeds back the index of chosen codeword \hat{i}_k to the BS [9]:

$$\mathbf{c}_{\hat{i}_k} = \arg \max_{\mathbf{c}_i \in \mathcal{C}} |\hat{\mathbf{g}}_{\Lambda_k}^H \mathbf{c}_i|^2. \quad (5)$$

Note that the mobile also feeds back $\|\mathbf{g}_{\Lambda_k}\|$ to the BS. Once the BS receives $\|\mathbf{g}_{\Lambda_k}\|$ and \hat{i}_k , the BS reconstructs the dominating PGI as $\hat{\mathbf{g}}_{\Lambda_k} = \|\mathbf{g}_{\Lambda_k}\| \mathbf{c}_{\hat{i}_k}$.

B. Dominating PGI Feedback-based Data Precoding

The data precoding vector \mathbf{w}_k for the k -th mobile, constructed from the dominating PGI feedback $\hat{\mathbf{g}}_{\Lambda_k}$, is given by

$$\mathbf{w}_k = \sqrt{\beta_k} \mathbf{A}_{\Lambda_k} \text{diag}(\mathbf{p}_{\Lambda_k}) \hat{\mathbf{g}}_{\Lambda_k} = \sqrt{\beta_k} \mathbf{A}_{\Lambda_k} \mathbf{P}_{\Lambda_k} \hat{\mathbf{g}}_{\Lambda_k}, \quad (6)$$

where $\mathbf{A}_{\Lambda_k} = [\mathbf{a}(\theta_{k,i}), \forall i \in \Lambda_k] \in \mathbb{C}^{N \times L}$ is the sub-matrix of \mathbf{A}_k , $p_{k,i} \in \mathbb{R}$ is the power weight for the i -th path, and $\mathbf{p}_{\Lambda_k} = [p_{k,i}, \forall i \in \Lambda_k]^T \in \mathbb{R}^L$. In the following proposition, we discuss the energy efficiency of proposed dominating PGI-based precoding scheme.

Proposition 1. *When the perfect dominating PGI is used, the energy efficiency of the mmWave massive MIMO systems is*

$$E_e = \frac{\sum_{k=1}^K \log_2 \left(1 + \frac{\beta_k^2 \left(|\mathbf{1}^T \mathbf{p}_{\Lambda_k}|^2 + \|\mathbf{A}_k^H \mathbf{A}_{\Lambda_k} \mathbf{P}_{\Lambda_k}\|_F^2 \right)}{\sum_{j \neq k} \beta_k \beta_j \|\mathbf{A}_k^H \mathbf{A}_{\Lambda_j} \mathbf{P}_{\Lambda_j}\|_F^2 + \sigma_n^2} \right)}{\frac{1}{\eta} \sum_{k=1}^K \beta_k \|\mathbf{p}_{\Lambda_k}\|^2 + NP_{\text{fix}}}. \quad (7)$$

¹ $\mathbf{W}_{p,k}$ is obtained from the multiplication of $1/\sqrt{\beta_k}$, $\mathbf{A}_k^+ = (\mathbf{A}_k^H \mathbf{A}_k)^{-1} \mathbf{A}_k^H$, and the binary grouping matrix \mathbf{G}_k such that $\mathbf{g}_{\Lambda_k} = \mathbf{G}_k \mathbf{g}_k$.

One can easily prove the proposition using [10, Theorem 5.2c]. The energy efficiency maximization problem to choose L dominating paths and the power weights is formulated as

$$\mathcal{P}_1 : \max_{\{\Lambda_k, \mathbf{p}_{\Lambda_k}\}} E_e \quad (8a)$$

$$\text{s.t. } |\Lambda_k| = L, \quad \forall k \in \mathcal{U} \quad (8b)$$

$$\frac{1}{\eta} \sum_{k=1}^K \beta_k \|\mathbf{p}_{\Lambda_k}\|^2 \leq P_{\text{max}}, \quad (8c)$$

where P_{max} is the maximum transmission power of BS and \mathcal{U} is the set of mobiles. We note that it is computationally infeasible to solve \mathcal{P}_1 since we need an exhaustive search over $({}^P C_L)^K$ candidates to find out the optimal solution.

C. Dominating Path Selection Problem Formulation

To solve \mathcal{P}_1 , we use the notion of *sparse power weight vector* $\mathbf{p}_k = [p_{k,1}, \dots, p_{k,P}]^T \in \mathbb{R}^P$ for all paths so that the dominating path indices $\{\Lambda_k\}$ and the power weights $\{\mathbf{p}_{\Lambda_k}\}$ are mapped to the positions and entries of nonzero elements of $\{\mathbf{p}_k\}$, respectively. In doing so, we can jointly optimize $\{\Lambda_k\}$ and $\{\mathbf{p}_{\Lambda_k}\}$ by solving the support identification problem for $\{\mathbf{p}_k\}$. Specifically, Λ_k is expressed as

$$\Lambda_k = \{i \mid |p_{k,i}| > 0, i = 1, \dots, P\} = \text{supp}(\mathbf{p}_k). \quad (9)$$

Then $|\Lambda_k| = L$ is converted to $\|\mathbf{p}_k\|_0 = L$. Also, using $\{\mathbf{p}_k\}$, E_e in (7) can be re-expressed as

$$E_e = \frac{\sum_{k=1}^K \log_2 \left(1 + \frac{|\mathbf{b}_k^T \mathbf{p}_k|^2 + \|\mathbf{C}_{k,k} \mathbf{p}_k\|^2}{\sum_{j \neq k} \|\mathbf{C}_{k,j} \mathbf{p}_j\|^2 + \sigma_n^2} \right)}{\frac{1}{\eta} \sum_{k=1}^K \beta_k \|\mathbf{p}_k\|^2 + NP_{\text{fix}}}, \quad (10)$$

where $\mathbf{C}_{k,j} = \sqrt{\beta_k \beta_j} \text{diag}(\|\mathbf{A}_k^H \mathbf{a}(\theta_{j,1})\|, \dots, \|\mathbf{A}_k^H \mathbf{a}(\theta_{j,P})\|)$ and $\mathbf{b}_k = \beta_k \mathbf{1}_P$. Then using the auxiliary variables $\{u_k, t_k\}$, we obtain the ℓ_0 -norm constrained optimization problem:

$$\mathcal{P}_2 : \max_{\{\mathbf{p}_k, u_k, t_k\}} \frac{\sum_{k=1}^K t_k}{t_0} \quad (11a)$$

$$\text{s.t. } \|\mathbf{p}_k\|_0 \leq L, \quad \forall k \in \mathcal{U} \quad (11b)$$

$$1 + \frac{|\mathbf{b}_k^T \mathbf{p}_k|^2 + \|\mathbf{C}_{k,k} \mathbf{p}_k\|^2}{\sum_{j \neq k} \|\mathbf{C}_{k,j} \mathbf{p}_j\|^2 + \sigma_n^2} \geq u_k, \quad \forall k \in \mathcal{U} \quad (11c)$$

$$\log_2(u_k) \geq t_k, \quad \forall k \in \mathcal{U} \quad (11d)$$

$$\frac{1}{\eta} \sum_{k=1}^K \beta_k \|\mathbf{p}_k\|^2 + NP_{\text{fix}} \leq t_0 \quad (11e)$$

$$\frac{1}{\eta} \sum_{k=1}^K \beta_k \|\mathbf{p}_k\|^2 \leq P_{\text{max}}. \quad (11f)$$

To solve \mathcal{P}_2 , one can use the re-weighted ℓ_2 -norm approximation [11] and successive convex approximation (SCA) [12].

First, using the re-weighted ℓ_2 -norm approximation, the ℓ_0 -norm of power weight vector is approximated as

$$\|\mathbf{p}_k\|_0 \approx \sum_{i=1}^P w_{k,i} |p_{k,i}|^2 = \|\mathbf{W}_k \mathbf{p}_k\|^2, \quad (12)$$

where $\mathbf{W}_k = \text{diag}(\sqrt{w_{k,1}}, \dots, \sqrt{w_{k,P}})$. Also, $w_{k,i}$ is the ℓ_2 -norm approximation weight which is iteratively updated as [11]

$$w_{k,i} = 1/(|p_{k,i}^{\text{past}}|^2 + \epsilon), \quad (13)$$

where $p_{k,i}^{\text{past}}$ is the power weight obtained in the previous iteration and $\epsilon > 0$ is a regularization factor. Since $w_{k,i}$ is inversely proportional to $|p_{k,i}^{\text{past}}|^2$, the transmit power would be focus on the path indices corresponding to the high power weights. Thus, as the iteration goes on, the sparsity of \mathbf{p}_k would decrease, ensuring the satisfaction of ℓ_0 -norm constraint (11b).

We then approximate the following rate function f to a linear function:

$$f(\mathbf{p}, u_k) = \frac{|\mathbf{b}_k^T \mathbf{p}_k|^2 + \sum_{j=1}^K \|\mathbf{C}_{k,j} \mathbf{p}_j\|^2 + \sigma_n^2}{u_k}. \quad (14)$$

The first-order Taylor expansion of f at the n -th iteration is

$$F(\mathbf{p}, u_k | \mathbf{p}^{(n)}, u_k^{(n)}) = f(\mathbf{p}^{(n)}, u_k^{(n)}) - \frac{u_k - u_k^{(n)}}{u_k^{(n)}} f(\mathbf{p}^{(n)}, u_k^{(n)}) + \frac{2}{u_k^{(n)}} \left(\mathbf{p}_k^{(n),T} \mathbf{b}_k \mathbf{b}_k^T (\mathbf{p}_k - \mathbf{p}_k^{(n)}) + \sum_{j=1}^K \mathbf{p}_j^{(n),T} \mathbf{C}_{k,j}^T \mathbf{C}_{k,j} (\mathbf{p}_j - \mathbf{p}_j^{(n)}) \right)$$

where $\mathbf{p}^{(n)} = \{\mathbf{p}_k^{(n)}\}$ and $u_k^{(n)}$ be the solutions obtained at the $(n-1)$ -th SCA iteration. Since F is a linear function of $\{\mathbf{p}_k^{(n)}\}$ and u_k , by approximating f into F in (11c), we obtain a convex second-order cone (SOC) constraint [13]. We also apply the same approximation to (11d) to obtain the SOC constraint. In addition, to convert the fractional objective function (11a) to a linear function, we use the Charnes-Cooper transformation [13]. Finally, the converted problem is modeled as a convex SOCP which can be solved by using the optimization solvers (e.g., Gurobi [14]).

Once the SCA iteration is finished, we update the ℓ_2 -norm approximation weights $\{w_{k,i}\}$ using (13) and then repeat these procedures until ℓ_0 -norm constraint (11b) is satisfied.

IV. NUMERICAL RESULTS

We consider the mmWave massive MIMO systems where a BS with $N = 64$ antennas serves $K = 4$ single antenna mobiles. The power consumption parameters of BS are $\eta = 0.6$, $P_{\max} = 2$ W, and $P_{\text{fix}} = 0.1$ W [15]. We use the multi-path channel model with $P = 20$ paths, carrier frequency 2 GHz, and channel bandwidth 10 MHz. The angle spread of AoD is 20° . The large-scale fading coefficient is given by $\beta_k = \text{PL}_k \times z_k$ where PL_k represents the three-slope path loss [16] and z_k represents the log-normal shadowing. In the proposed scheme, we choose $L = 4$ dominating paths. For comparison, we use the two conventional schemes: 1) CSI feedback-based scheme [17] using 4-bit channel statistic codebook [18] and 4-bit RVQ codebook [9] and 2) TDD-based scheme [19].

In Fig. 2, we plot the energy efficiency as a function of SNR. We plot the performances of ideal systems with perfect

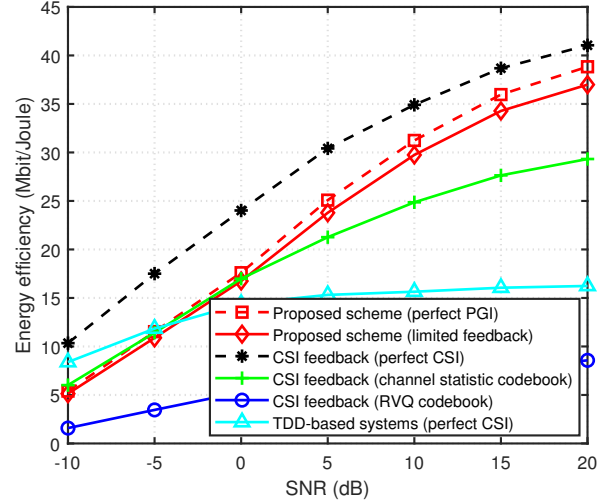


Fig. 1: Energy efficiency as a function of transmit SNR

channel information and realistic systems with finite rate feedback as a dotted line and a real line, respectively. We observe that the proposed scheme significantly improves the energy efficiency over the conventional schemes. For example, when SNR = 20 dB, the energy efficiency gains of proposed scheme over the channel statistic codebook scheme and the RVQ codebook scheme are almost 22% and 331%, respectively. The reason is that the conventional scheme feeds back the full-dimensional channel vector, whereas the proposed scheme feeds back the dominating PGI vector with reduced dimension. Note that in the TDD-based systems, due the lack of downlink training procedure, instantaneous CSI is not available at the mobile so that the detection and decoding error caused by the channel estimation error is unavoidable.

In Fig. 3, we plot the energy efficiency as a function of feedback bits B when SNR = 16 dB. We observe that the proposed scheme reduces more than 75% feedback overhead over the conventional power allocation schemes. For instance, while the conventional scheme requires $B = 8$ bits to achieve the energy efficiency of 32 Mbit/Joule, the proposed scheme requires only $B = 2$ bits. Also, we observe that when $B = 2$, the energy efficiency gap of the proposed scheme is 3.1 Mbit/Joule whereas those of conventional channel statistic-based codebook scheme and the RVQ codebook scheme are 12.7 Mbit/Joule and 33.5 Mbit/Joule, respectively.

V. CONCLUSION

In this paper, we proposed an energy efficiency maximization technique for mmWave massive MIMO systems using sparse channel feedback. Key idea of the proposed scheme is to choose a few dominating paths in the angular domain and then only feed back the PGIs of chosen paths to BS. Since the number of dominating paths is much smaller than the number of antennas, the feedback overhead is reduced substantially. From the energy efficiency analysis in the limited feedback scenario, we showed that the energy efficiency loss of the proposed scheme is much smaller than that of the conventional

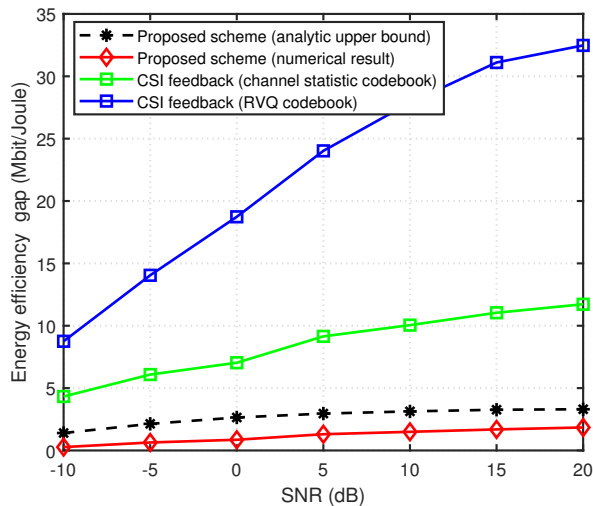


Fig. 2: Energy efficiency gap as a function of transmit SNR

CSI feedback-based scheme. Also, from the simulation results, we showed that the proposed scheme achieves a significant energy efficiency improvement over the conventional schemes.

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