Quantum Teleportation: A Premier Quantum Communication Protocol

Chaemoon Im†, Soohyun Park§, and Joongheon Kim†
†Department of Electrical and Computer Engineering, Korea University, Seoul, Republic of Korea
§Division of Computer Science, Sookmyung Women's University, Seoul, Republic of Korea
E-mail: anscodla0314@korea.ac.kr, soohyun.park@sookmyung.ac.kr, joongheon@korea.ac.kr

***Abstract*—Communication is one of the fields that adopts quantum characteristics to improve its potential. In particular, quantum teleportation (QT) is considered a fundamental tool for enabling quantum networks. This paper focuses on the concept of QT and how it is applied in conjunction with additional technologies such as entanglement purification, entanglement swapping, and quantum error correction.**

I. Introduction

Quantum technology aims to solve problems that classical computing technologies cannot, by adopting quantum characteristics such as superposition and entanglement [1]. Among various fields, communication is one of the areas where quantum characteristics are actively adopted, yielding technologies such as quantum key distribution (QKD) [2] and quantum teleportation (QT) [3]. In particular, QT plays a major role in quantum communication as a fundamental protocol for sending arbitrary quantum states to another quantum system. Furthermore, based on QT, research on quantum internet is being conducted, aiming to connect quantum devices at different locations, creating a quantum network [4]. This paper addresses the concept of QT and various techniques for creating quantum networks. Moreover, this paper suggests future directions for quantum networking.

II. Quantum Teleportation

Fig. 1 illustrates a quantum circuit for QT. QT is a quantum communication protocol designed to transmit a quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ from sender Alice to receiver Bob. To deliver the quantum state, they use an entangled pair of qubits, typically in the Bell state $|\Phi^+\rangle$, distributing one qubit to each of them. The entire system ρ, including $|\psi\rangle$ and $|\Phi^+\rangle$, can be described as $\rho = |\psi\rangle|\Phi^+\rangle$.

Between the computational basis $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$ and the Bell basis $|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle$, and $|\Psi^-\rangle$, the following relation holds, i.e.,

$$\begin{cases} |00\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle), \\ |01\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle), \\ |10\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle), \\ |11\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle). \end{cases} \tag{1}$$

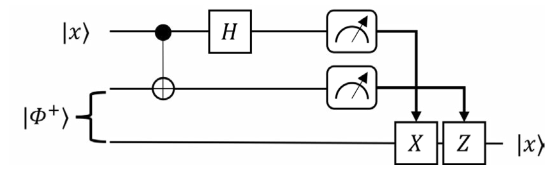


Fig. 1: A quantum circuit for QT.

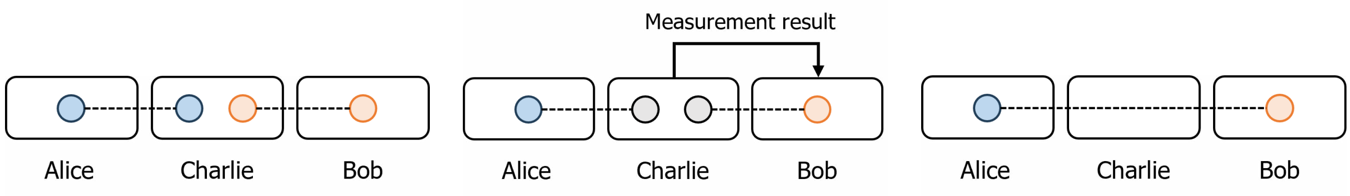
Using above equation, ρ can be expressed as follows,

$$\rho = \frac{1}{2}(|\Phi^+\rangle\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{|\phi_0\rangle} + |\Psi^+\rangle\underbrace{(\beta|0\rangle + \alpha|1\rangle)}_{|\phi_1\rangle} + |\Psi^-\rangle\underbrace{(\alpha|0\rangle - \beta|1\rangle)}_{|\phi_2\rangle} + |\Phi^-\rangle\underbrace{(\beta|0\rangle - \alpha|1\rangle)}_{|\phi_3\rangle}). \tag{2}$$

It can be observed that by performing a Bell basis measurement from Alice, the original 3-qubit system ρ collapses to one of the states among $|\phi_0\rangle = |\psi\rangle, |\phi_1\rangle = X|\psi\rangle, |\phi_2\rangle = Z|\psi\rangle$ and $|\phi_3\rangle = XZ|\psi\rangle$. Thus, Bob can apply a certain quantum gate to the collapsed states to restore the original state $|\phi\rangle$. For example, if the measurement result is 10, Bob applies a Z gate to the collapsed states to negate the sign.

From the point of view of communication, Alice's measurement result acts as an indicator that measures how the original state $|\psi\rangle$ is perturbed. Similarly, applying quantum gates to Bob's qubit can be analyzed as a decoding process for a qubit that has been modified by a quantum channel. It is also evident that Bell basis measurement of Alice can be considered as an encoding process.

After its first discovery by [1] C. H. Bennett et al, numerous adaptations and modifications have been applied to QT to further expands its possibilities for the quantum communication protocols. V. Verma [2] modifies QT to bidirectional QT, enabling bidirectional communication between two quantum devices. Furthermore, D.Song et al, [3] enhances the capacity of QT by sending multiple qubits. Moreover, sending classical information using QT is also under active research. Especially, M. Karthik et al [4] succeeds sending image or audio via QT by incorporating QT with Huffman coding [5].



(a) Initial state before swapping. (b) Bell basis measurement by Charlie. (c) Entanglement swapping.

Fig. 2: A procedure of entanglement swapping. In (a), Alice and Bob are sharing its Bell pair with intermediate node, Charlie. In (b), Charlie performs Bell basis measurement to its qubits and send its measurement result to Bob. Lastly, in (c), entanglement between Alice and Bob is created, and Bob initializes entanglement state by using the measurement result from Charlie.

III. Quantum Networking based on QT

To incorporate QT into networking between quantum devices, QT has been expanded and applied to connect different nodes in the network. Technologies for quantum networking, including entanglement swapping [6] and entanglement purification [7], are addressed in this section.

A. Entanglement Swapping

Entanglement swapping is used to create an entanglement between sender Alice and receiver Bob, who each shares entangled pair of qubits, typically Bell state $|\Phi^+\rangle$, with intermediate node, Charlie, as illustrated in Fig. 2 (a). The entire system ρ before entanglement swapping can be descried as $\rho = |\Phi^+\rangle|\Phi^+\rangle$.

By Eq. 1, ρ can be further unfolded as follows,

$$\rho = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle) \tag{3}$$

$$= \frac{1}{2}(|00\rangle_{\mathrm{AB}}|00\rangle_{\mathrm{C}} + |01\rangle_{\mathrm{AB}}|01\rangle_{\mathrm{C}} \tag{4}$$

$$+ |10\rangle_{\mathrm{AB}}|10\rangle_{\mathrm{C}} + |11\rangle_{\mathrm{AB}}|11\rangle_{\mathrm{C}}) \tag{5}$$

$$= \frac{1}{2}(|\Phi^+\rangle_{\mathrm{AB}}|\Phi^+\rangle_{\mathrm{C}} + |\Phi^-\rangle_{\mathrm{AB}}|\Phi^-\rangle_{\mathrm{C}} \tag{6}$$

$$+ |\Psi^+\rangle_{\mathrm{AB}}|\Psi^+\rangle_{\mathrm{C}} + |\Psi^-\rangle_{\mathrm{AB}}|\Psi^-\rangle_{\mathrm{C}}). \tag{7}$$

By the Bell basis measurement of Charlie, the entanglement between Alice and Bob can be made, as illustrated in Fig.2 (b). Furthermore, sender and receiver should initialize their channel qubit as they want, using the measurement result of the intermediate node, Charlie. For example, if Charlie's measurement result is 01, state of the qubit is $|\Psi^+\rangle$, thus Bob can apply X gate to his qubit, to modify entangled state as the type Alice and Bob wants. This initialization process is illustrated in Fig.2 (c).

Entanglement swapping enables sender and receiver at a long distance but not having entangled pair, to communicate using other connected nodes. It can be used to create quantum repeater, which is a device that relays different nodes in the quantum network, by successively performing entanglement swapping between nodes. Note that the intermediate node that has connection with multiple nodes can act as a router, which has a fundamental role for utilizing quantum networks.

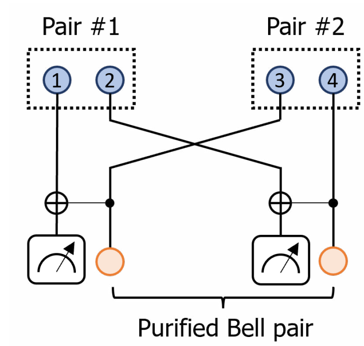


Fig. 3: An illustration of the entanglement purification process.

B. Entanglement Purification

To maintain the quality of entanglement between nodes and ensure reliable communication using QT, entanglement purification is necessary. Entanglement purification creates highly entangled state using multiple less entangled states, which was first proposed by Bennett *et al.* [7].

Assume that there are 2 Bell pair, which are affected by X error with probability of p. Then, state of these bell pairs ρ can be described as an ensemble state of $\rho = (1-p)|\Phi^+\rangle + p|\Psi^+\rangle$, since $|\Psi^+\rangle = I \otimes X|\Phi^+\rangle$.

According to the procedure of Fig. 3, quantum state before applying CNOT gates can be described as follows,

$$\rho' = \rho^{\otimes 2} = (1-p)^2|\Phi^+\rangle_{12}|\Phi^+\rangle_{34} + p(1-p)|\Phi^+\rangle_{12}|\Psi^+\rangle_{34} + p(1-p)|\Psi^+\rangle_{12}|\Phi^+\rangle_{34} + p^2|\Psi^+\rangle_{12}|\Psi^+\rangle_{34}. \tag{8}$$

After swapping second, third qubit and performing CNOT gates, the entire state is divided into two parts, which are purified component and mixed component.

$$\rho' = (1-p)^2 \underbrace{|\Phi^+\rangle_{13}|\Phi^+\rangle_{24}}_{\texttt{purified}} + 2p(1-p)\underbrace{|\Psi^+\rangle_{13}|\Phi^+\rangle_{24}}_{\texttt{mixed}} + p^2 \underbrace{|\Psi^+\rangle_{13}|\Psi^+\rangle_{24}}_{\texttt{purified}}. \tag{9}$$

It is possible to discriminate those states by performing measurement on first and second qubits. By the property of $|\Phi^+\rangle$ and $|\Psi^+\rangle$, the measurement results of two qubits are always same when it was $|\Phi^+\rangle$ and always different when it was $|\Psi^+\rangle$.

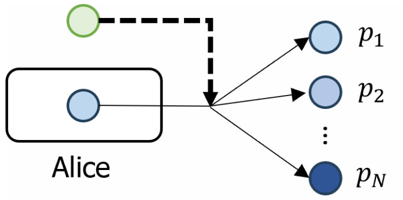


Fig. 4: An illustration the effect of quantum channel. Green dot represents environment qubit which models interaction with the environment.

Thus, if the measurement result of two qubit is different with a probability of $2p(1-p)$, it is possible to achieve pure Bell state $|\Phi^+\rangle$ with probability of $(1-p)^2/((1-p)^2+p^2)$, by disregarding the first Bell pair. If the measurement result is same, it is possible to get pure Bell pair.

IV. Reliable Quantum Communication

In Noisy-Intermediate-Scale Quantum (NISQ) era, the error from the quantum channel is inevitable due to the imperfect gate operation or imperfect state preparation, which weakens the entanglement of the communication resource and degrades the performance of the QT. Analyze on this error is essential to prevent QT from affecting from the noise and enable reliable communications between nodes.

A. Quantum Channel

In many cases, quantum system is considered as closed system, which means that there is no interaction with other system. However, interactions with other system during quantum information processing cannot be avoided. In this sense, quantum channel $\mathcal{E}(\rho)$ can be analyzed as 'environment' qubit that interacts with the system's qubit [8]. These environment qubit models the interactions of the system with the environment, making input quantum state $|\psi\rangle$ into ensemble state $|\psi'\rangle = \{|\psi_i\rangle, p_i\}$, as illustrated in Fig. 4. Among several channel, amplitude damping channel, phase damping channel, and depolarizing channel are usually adopted to model real-world phenomenons [9]. These models represents physical phenomenons of quantum information processing system, such as spontaneous energy emission or decoherence [9], which acts as a hinder for the quantum communication.

B. Quantum Error Correction

Many studies are being conducted in the field of quantum error correction (QEC), to precisely analyze the effect of the quantum channel which occurs during the quantum information processing, and devise a solution for mitigating those errors [10].

To address the problem from the quantum channel, conventional error detection and correction schemes for classical communication is naturally introduced to the quantum field. One of the basic approach for QEC is creating a logical qubit, which is tolerant to various quantum errors. One of the basic form of logical qubit is 3-qubit bit flip code, which can be defined as $|\psi\rangle_L = \alpha|000\rangle + \beta|111\rangle$ [10]. In this case, redundant 2 qubit provide information about the original qubit. Compared to bit flip code, Bacon-Shor code (BS code) [11] and surface code [12] are recently developed in the fields of QEC. These codes encode one logical qubit as a form of 2D lattice, while providing a capability to be tolerant at X and Z errors.

V. Conclusion

QT has been continuously expanded itself, in terms of increasing the number of participants in the network or increasing the capacity of QT. Furthermore, basic technologies for creating quantum networks have also been developed, motivating the utilization of the quantum network. However, there are still remaining challenges on QT, which should be dealt with to further utilize it. In particular, anonymity and privacy of the quantum network should be addressed, since quantum communication technologies are at its early stages.

Acknowledgement

This research was supported by the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2024-RS-2024-00436887) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation). J. Kim is the corresponding author of this paper(joongheon@korea.ac.kr).

References

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels," *Phys. Rev. Lett.*, vol. 70, no. 13, p. 1895, Mar. 1993.

[2] V. Verma, "Bidirectional quantum teleportation by using two ghz-states as the quantum channel," *IEEE Communications Letters*, vol. 25, no. 3, pp. 936–939, Nov. 2021.

[3] D. Song, C. He, Z. Cao, and G. Chai, "Quantum teleportation of multiple qubits based on quantum fourier transform," *IEEE Communications Letters*, vol. 22, no. 12, pp. 2427–2430, Oct. 2018.

[4] M. Karthik, J. Lalwani, and B. Jajodia, "Quantum image teleportation protocol (QITP) and quantum audio teleportation protocol (QATP) by using quantum teleportation and huffman coding," in *Proc. Int'l Conf. on Trends in Quantum Computing and Emerging Business Technologies (TQCEBT)*, Maharashtra, India, Oct. 2022, pp. 1–6.

[5] T. M. Cover, *Elements of information theory*. John Wiley & Sons, 1999.

[6] A. H. Abbas, Y. S. Mudhafar, A. T. Kareem, and W. A. Abohana, "Quantum broadcast by entanglement swapping," in *Proc. Int'l Conf. on Smart Applications, Communications and Networking (SmartNets)*, Harrisonburg, Washington DC, May. 2024, pp. 1–5.

[7] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, "Purification of noisy entanglement and faithful teleportation via noisy channels," *Phys. Rev. Lett.*, vol. 76, pp. 722–725, Jan. 1996.

[8] O. Simeone, "An introduction to quantum machine learning for engineers," *Found. Trends Signal Process.*, vol. 16, no. 1-2, pp. 1–223, Jul. 2022.

[9] R. Fortes and G. Rigolin, "Fighting noise with noise in realistic quantum teleportation," *Physical Review A*, vol. 92, no. 1, p. 12338, Jul. 2015.

[10] S. J. Devitt, W. J. Munro, and K. Nemoto, "Quantum error correction for beginners," *Reports on Progress in Physics*, vol. 76, no. 7, p. 76001, Jun. 2013.

[11] P. Aliferis and A. W. Cross, "Subsystem fault tolerance with the bacon-shor code," *Phys. Rev. Lett.*, vol. 98, p. 220502, May 2007.

[12] A. G. Fowler, D. S. Wang, C. D. Hill, T. D. Ladd, R. Van Meter, and L. C. L. Hollenberg, "Surface code quantum communication," *Phys. Rev. Lett.*, vol. 104, p. 180503, May. 2010.