Performance Analyses of Multiple Antennas Based Sequential Detection for Unknown Signals

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***Abstract*—Sequential detection is one of the signal detection technique which can reduce the number of required samples for signal detection. However, prior knowledge for target signal power is required in sequential detection. In this study, the performances of multiple antennas based sequential detection for the power of unknown signals are investigated. Numerical examples show the effectiveness of the analytical results.**

***Index Terms*—Sequential detection, signal detection, multiple antennas**

I. Introduction

Recently, the signal detection techniques are widely employed to several wireless communication systems, e.g., spectrum sensing in cognitive radio [1]. Various signal detection techniques have been studied, energy detection [2], cyclostationaly detection [3], matched filter based detection [4], and so on. Various applications of these signal detection techniques are begin considered [5], [6]. One common feature of these signal detection techniques is that samples are acquired during a fixed signal detection period, and the statistics computed from them are used to decide whether the target signal is present or absent. This leads that a fixed computational cost is required for signal detection even in the case where it is clear that the target signal is present. The reduction of the computational cost in signal detection must be achieved in battery-powered receivers. Sequential detection [7] is one of the signal detection technique that the statistics are sequentially computed and signal detection is executed at each time a sample is acquired. Conventionally, sequential detection techniques based on cooperative systems [8] and multiple antennas based [9] have been presented.

However, the prior knowledge of the target signal power is fundamentally required in sequential detection, and this is an unrealistic assumption in the actual environment. Although sequential detection for unknown target signal power is discussed [8], the characteristics of sequential detection for the case have never been revealed. In this study, the characteristics of multiple antennas based sequential detection for the unknown target signal power are investigated.

II. Preliminary Notions

The signal detection problem can be considered as a binary hypothesis testing problem. We let $\mathcal{H}_0$ and $\mathcal{H}_1$ denote hypotheses which indicate the target signal is present and absent, respectively. Assuming that the receiver has N_R antennas and RF chains, the received signal at the kth antenna $r_k(n)$ can be expressed by

$$\begin{array}{l}\mathcal{H}_1 : r_k(n) = h_k s(n) + v_k(n)\\ \mathcal{H}_0 : r_k(n) = v_k(n)\end{array}, \qquad k = 1, \cdots, N_\mathrm{R}, \tag{1}$$

where h_k, $s(n)$, and $v_k(n)$ are the complex gain between the target and kth antenna, the target signals, and additive white Gaussian noise (AWGN) at the kth antenna with variance $\sigma^2_{v,k}$, respectively. Note that $s(n)$ and $v_k(n), \forall k$ are statistically independent.

In sequential detection, the statistics are sequentially computed at each time a sample is obtained, and the decision for signal detection is executed at each time. Therefore, the number of samples for signal detection can be reduced. The statistics $LLR(\nu)$ at the νth detection based on log-likelihood test for multiple antennas based sequential detection can be written by

$$\begin{aligned} LLR(\nu) &= \ln\left(\prod_{n=1}^{\nu}\prod_{k=1}^{N_\mathrm{R}} \frac{p_{1,k}\left(r_k(n)\right)}{p_{0,k}\left(r_k(n)\right)}\right) \\ &= \sum_{n=1}^{\nu}\sum_{k=1}^{N_\mathrm{R}}\left[\left(\frac{1}{2\sigma^2_{0,k}} - \frac{1}{2\sigma^2_{1,k}}\right) r_k^2(n) - \frac{1}{2}\ln\frac{2\sigma^2_{1,k}}{2\sigma^2_{0,k}}\right] \\ &\qquad \nu = 1, 2, \cdots \end{aligned} \tag{2}$$

where $p_{1,k}\left(r_k(n)\right)$, $p_{0,k}\left(r_k(n)\right)$, $\sigma^2_{1,k}$, and $\sigma^2_{0,k}$ are likelihood of $r_k(n)$ in $\mathcal{H}_1$, likelihood of $r_k(n)$ in $\mathcal{H}_0$, the lower bound for variance $r_k(n)$ in $\mathcal{H}_1$, and the lower bound for variance $r_k(n)$ in $\mathcal{H}_0$, respectively. In the case that the power of target signal is unknown, the assumed lower bound of received power $\widetilde{\sigma_{1,k}}^2$ is employed to $\sigma^2_{1,k}$ and the variance of $v_k(n)$ is employed to $\sigma^2_{0,k}$. Signal detection based on eq. (2) is executed as

$$\begin{array}{rl} B < LLR(\nu) < A : & \text{Go on} \\ LLR(\nu) \leq B : & \text{Accept } \mathcal{H}_0 \\ LLR(\nu) \geq A : & \text{Accept } \mathcal{H}_1 \end{array} \tag{3}$$

where $A = \ln\{(1-\beta)/\alpha\}$ and $B = \ln\{\beta/(1-\alpha)\}$ are thresholds, respectively, and α and β are the upper bounds of false alarm probability and miss detection, respectively.

III. Performance Analyses

The purpose of this study is to discuss the performances of multiple antennas based sequential detection in the case that the power of target signal is unknown. We let γ denote that an assumed lower bound of signal power to noise ratio (SNR), and γ is given by

$$\gamma = 10\log_{10}\left(\frac{\widetilde{\sigma_{1,k}}^2}{\sigma_v^2}\right). \tag{4}$$

where $\sigma_v^2 = E\left[\sigma_{v,k}^2\right]$. The performances of multiple antennas based sequential detection are evaluated for several γs. In the evaluation, actual SNR of the target signal for each antenna is randomly selected from γ to 0, and averaged characteristics for $500,000$ trials are presented. Furthermore, the white Gaussian signal is employed to the target signal and $N_\mathrm{R} = 4$ is employed.

Figs. 1 and 2 show the number of required samples for signal detection and the false alarm and miss detection probabilities, respectively. In both figures, characteristics for different α, β and $\gamma = -5\,\mathrm{dB}$, $-10\,\mathrm{dB}$, $-15\,\mathrm{dB}$ are depicted. As shown in Fig. 1, the average number of required samples increases as γ decreases. Furthermore, as shown in Fig. 2, the miss detection probability deteriorates as γ decreases. To reveal their characteristics, the possible range of $LLR(\nu)$ is investigated. Fig. 3 shows the maximum and minimum values of $LLR(\nu)$ for different α, β, and γ. It can be seen that the possible range of $LLR(\nu)$ is wider as γ increases. The reason can be explained as follows. In the case of small γ, the fluctuation range of $LLR(\nu)$ becomes smaller than the threshold because the power of target signal is relatively small. As a result, the accuracy of signal detection can be improved although the greater number of samples is required. On the other hand, in the case of large γ, the fluctuation range of $LLR(\nu)$ becomes larger than the threshold because the power is relatively large. As a result, the smaller number of samples is required although the miss detection probability deteriorates.

IV. Conclusion

In this study, we investigated the performances of multiple antennas based sequential detection for the power of unknown signals. Concretely, the tradeoff between the number of required samples and the miss detection probability were revealed. Numerical examples showed that the difference between the assumed lower bound of SNR determines and actual SNR determines the performances of multiple antennas based sequential detection.

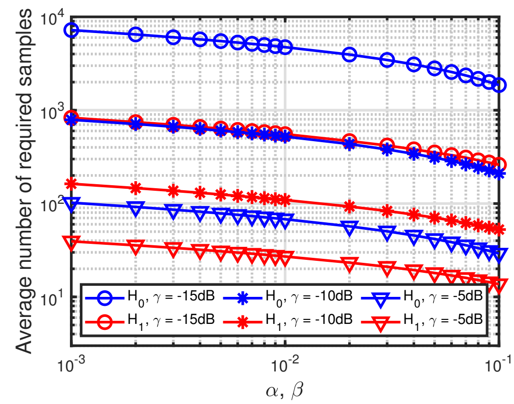

Fig. 1. Characteristics of average number of required samples for signal detection.

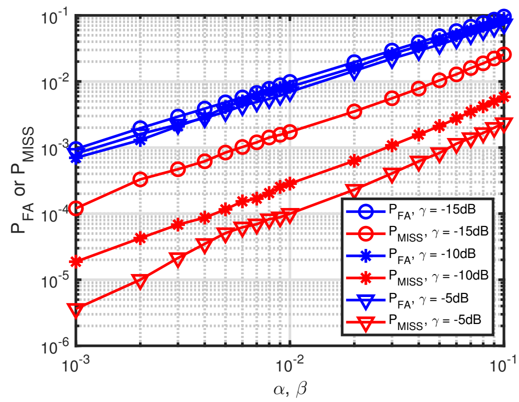

Fig. 2. Characteristics of false alarm probability and miss detection probability.

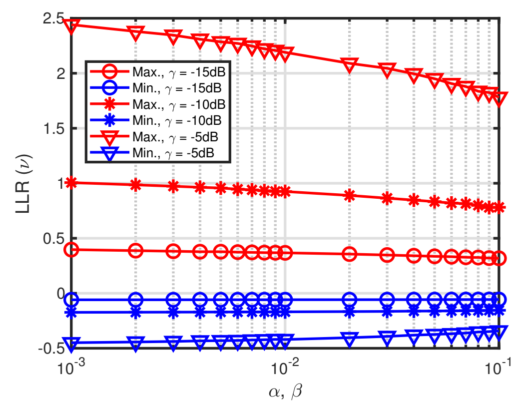

Fig. 3. Maximum and minimum values of $LLR(\nu)$.

References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005.

[2] H. Urkowitz, "Energy Detection of Unknown Deterministic Signals," *Proc. of the IEEE*, vol. 55, no. 4, pp. 523–531, 1967.

[3] W. Gardner, "Exploitation of Spectral Redundancy in Cyclostationary Signals," *IEEE Signal Processing Magazine*, vol. 8, no. 2, pp. 14–36, 1991.

[4] R. Tandra and A. Sahai, "Fundamental Limits on Detection in Low SNR Under Noise Uncertainty," in *2005 International Conference on Wireless Networks, Communications and Mobile Computing*, vol. 1, 2005, pp. 464–469 vol.1.

[5] D. Cho and S. Narieda, "Simple Weighted Diversity Combining Technique for Cyclostationarity Detection Based Spectrum Sensing in Cognitive Radio Networks," *IEICE Transaction on Communications*, vol. E99-B, no. 10, pp. 2212–2220, Oct. 2016.

[6] S. Narieda and T. Fujii, "Energy Detection Based Carrier Sense in LPWAN," *IEEE Access*, vol. 11, pp. 79 105–79 119, 2023.

[7] A. Wald, *Sequential Analysis*. John Wiley and Sons, 1947.

[8] Q. Zou, S. Zheng, and A. H. Sayed, "Cooperative Sensing via Sequential Detection," *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6266–6283, 2010.

[9] R. Hayashi, S. Narieda, and H. Naruse, "Sequential Spectrum Sensing Based on Multiple Antennas with Low Computational Complexity Statistic," *IEICE Communication Express*, vol. 9, no. 11, pp. 541–546, 2020.