Orthogonal Time Frequency Space (OTFS) With Tri-Mode Index Modulation

Adnan Tariq, Muhammad Sajid Sarwar, Soo Young Shin, *Senior Member, IEEE*

***Abstract*—Orthogonal time frequency space with index modulation (OTFS-IM) has been considered a promising technique for improving the bit error rate (BER) in high mobility environments. To further improve the BER and spectral efficiency, here, tri-mode index modulation is proposed with OTFS modulation (TM-OTFS-IM). A portion of information bits is modulated with two distinguishable constellations, and the rest of the delay Doppler resource blocks (DDRBs) remain inactive in the DDRB grid. Through this approach, extra bits can be transmitted by the DDRB indices that correspond to the two constellation sets. The TM-OTFS-IM presented has the ability to improve both the spectrum and energy efficiency when compared to previous OTFS-IM, dual mode OTFS-IM and OTFS methods. Simulation results show the reduction in BER using TM-OTFS-IM as compared to DM-OTFS-IM and traditional OTFS-IM.**

***Index Terms*—Index modulation (IM), OTFS-IM, Tri-Mode OTFS.**

I. Introduction

Researchers are seeking novel and supplementary methods to address the growing need for high-speed mobile wireless communications via channels with limited bandwidth [1]–[3]. The orthogonal time frequency space (OTFS) modulation technique is a highly promising option for reducing the bit error rate (BER) in highly mobile environments, in comparison to traditional orthogonal frequency division multiplexing (OFDM) [2]. This is because OTFS utilizes delay and Doppler parameters to modulate information symbols, making it a robust choice for channels that are selective in both time and frequency [4].

In recent years, index modulation (IM) approaches have made significant contributions to the comprehension of spectrum efficiency, energy efficiency, and bit error rate (BER) [5], [6]. In index modulation (IM), the indices of transmission resources—including Delay Doppler Resource Blocks (DDRB), transmit antennas, subcarriers, time slots, and RF mirrors—can be employed to transmit supplementary information [7], [8]. The transmitted data block is generally segmented into two distinct parts. The first segment undergoes conventional modulation, while the second segment activates a subset of the transmission resources. The Generalized Index Modulation (GIM) scheme for MIMO-OTFS transmission is presented in [9] which leverages the spatial, delay, and Doppler domain indices to convey additional information, thereby improving spectral efficiency. The authors introduces a two-stage detection method to reduce the high complexity associated with maximum likelihood detection in this context. To enhance spectral efficiency (SE) and energy efficiency (EE) in OTFS by using multiple modulation modes across different grid resources in the delay-Doppler domain, a scheme is proposed in [10] which allows all grid points to be activated, transmitting both modulation bits and index bits implicitly through the combination of different constellation modes. Here a distance-based signal detection algorithm is also introduced to recover the transmitted information efficiently.

The Dual Mode OTFS-IM technique has been presented in [11] in which some delay Doppler (DD) resources are activated with one modulation order and the rest of the resources are utilised with another constellation.The constellation for each symbol is decided by the pattern of incoming bits. Dual Mode OTFS aims to increase the spectral efficiency by utilizing the inactive resources of conventional OTFS-IM at a fair cost of BER. To further increase the BER a new technique was introduced as Zero Padded Tri mode OFDM index modulation (ZTM-OFDM-IM) [12]. The ZTM-OFDM-IM uses two different constellations for modulating the information symbols and some resources are still empty to mitigate the inter-symbol interference. In this way ZTM-OFDM-IM improves the BER as compare to Dual Mode index modulation and improves the spectral efficiency as compare to traditional index modulation.

To harness the advantages of both techniques, such as using different modulation orders for information symbols and leaving some resources inactive, in highly mobile environments, this paper presents Tri-Mode Orthogonal Time Frequency Space Index Modulation (TM-OTFS-IM). The technique involves division of Delay Doppler Resource into sub-blocks. Within each OTFS sub-block, a portion of DD resources are modulated using two distinct constellation alphabets, while the other DD resources are left unmodulated. Data symbols can carry information bits and activated DD resources can likewise convey information through their indices. At the receiver, demodulation is performed using maximum-likelihood (ML) detector.

II. System Model

Fig. 1 presents the down-link system model for the proposed index Modulation assisted TM-OTFS. The transmitting information bits, B, are divided into G groups containing b_{TM} bits. In each group, b_{TM} bits are further divided into b_1 modulation bits and b_2 Index bits, where b_1 bits are modulated over the active DD resources pattern decided by the incoming index

Adnan Tariq, Muhammad Sajid Sarwar, and Soo Young Shin are affiliated with the Department of IT Convergence Engineering, Kumoh National Institute of Technology, Gumi, South Korea. Email: {adnantariq, sajid.sarwar, wdragon}@kumoh.ac.kr

*Corresponding Author: Soo Young Shin (wdragon@kumoh.ac.kr).

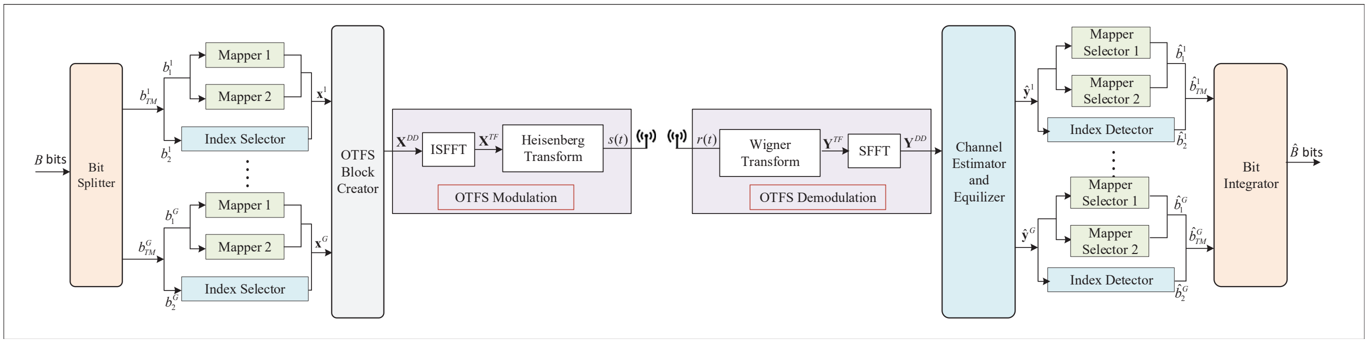

Fig. 1: System Model for proposed Index Modulation Assisted Tri-Mode OTFS (TM-OTFS-IM)

TABLE I: An example of a lookup table for $a = 2, a_1 = 1, a_2 = 1$ and $n = 4$.

Index Bits	Activation Pattern
000	$[\xi(\mathcal{M}_1), \xi(\mathcal{M}_2), 0, 0]$
001	$[\xi(\mathcal{M}_1), 0, \xi(\mathcal{M}_2), 0]$
010	$[\xi(\mathcal{M}_1), 0, 0, \xi(\mathcal{M}_2)]$
011	$[0, \xi(\mathcal{M}_1), \xi(\mathcal{M}_2), 0]$
100	$[0, \xi(\mathcal{M}_1), 0, \xi(\mathcal{M}_2)]$
101	$[0, 0, \xi(\mathcal{M}_1), \xi(\mathcal{M}_2)]$
110	$[\xi(\mathcal{M}_2), \xi(\mathcal{M}_1), 0, 0]$
111	$[\xi(\mathcal{M}_2), 0, \xi(\mathcal{M}_1), 0]$

bits b_2. Out of total MN delay Doppler resource blocks, where M is the number of resources along the delay axis and N is the number of resources along the Doppler axis, each group contains $n = MN/G$ DDRBs. The first bits $b_2 = \left\lfloor \log_2 \left(\binom{n}{a} \times \binom{a}{a_1} \right) \right\rfloor$ are utilized to choose a specific arrangement of a active DDRB from a total of n, with the remaining $n-a$ DDRB being assigned a value of zero. Here a_1 and $a_2 = a - a_1$ DDRBs are used for modulation $\mathcal{M}_1$ (Mapper 1) and $\mathcal{M}_2$ (Mapper 2) respectively. The active DDRBs indices for each group can be written as

$$I = [i_1, i_2, \ldots, i_a],$$

An example of arrangement of active DDRB and distinct modulations is shown in the TABLE 1. The $b_1 = a_1 \log_2(\mathcal{M}_1) + a_2 \log_2(\mathcal{M}_2)$ modulation bits are modulated on two different constellations defined by $\mathcal{M}_1$ and $\mathcal{M}_2$. An example of constellation symbols for $\mathcal{M}_1 = 8$ and $\mathcal{M}_2 = 2$ is hown in Fig.2. Now the total number of information bits that can be transmitted by the proposed index modulation assisted TM-OTFS are given by $b_{TM} = b_1 + b_2$

$$b_{TM} = a_1 \log_2(\mathcal{M}_1) + a_2 \log_2(\mathcal{M}_2) + \left\lfloor \log_2 \left(\binom{n}{a} \times \binom{a}{a_1} \right) \right\rfloor \quad (1)$$

Using the incoming bits for index mapping as illustrated in Table I, the g-th subblock can be represented as follows:

$$\mathbf{x}^g = [x^g(1), \ldots, x^g(\zeta), \ldots, x^g(n)]^{\mathrm{T}}, \; 1 \leq g \leq G, \quad (2)$$

where $x^g(\zeta) \in \{0, \xi(\mathcal{M}_1), \xi(\mathcal{M}_2)\}$. Here $\xi(\mathcal{M}_1), \xi(\mathcal{M}_2)$ represents the modulated symbols with respect to constellation

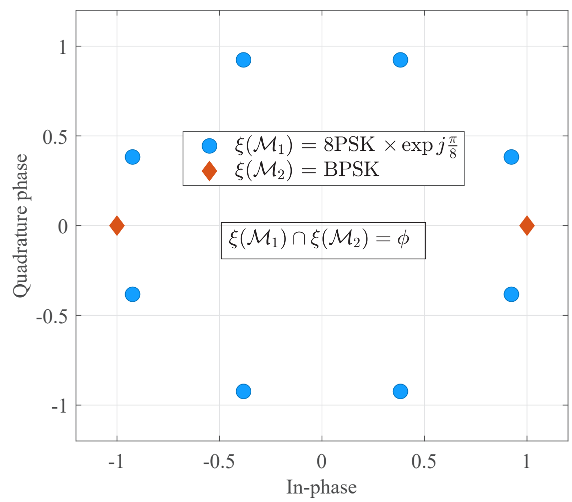

Fig. 2: An example of constellation symbols for $\mathcal{M}_1 = 8$ and $\mathcal{M}_2 = 2$.

order of $\mathcal{M}_1, \mathcal{M}_2$ respectively. Subsequently, all the sub-blocks are processed through the OTFS Block generator to produce $M \times N$ transmitting matrix $\mathbf{X}^{DD}$ in DD domain.

$$\mathbf{X}^{DD}[l,k] = \begin{pmatrix} x_{1,1} & \cdots & x_{N,1} \\ \vdots & \ddots & \vdots \\ x_{M,1} & \cdots & x_{M,N} \end{pmatrix} \quad (3)$$

where $l = 0, \cdots, M-1, k = 0, \cdots, N-1$. To convert the symbols into the time domain for transmission over a physical wireless channel, the Delay-Doppler (DD) domain symbols $x[k,l]$ are first transformed into Time-Frequency (TF) domain symbols $X[n,m]$ using the Inverse Symplectic Finite Fourier Transform (ISFFT). Subsequently, the Heisenberg transform is employed to convert the TF domain symbols into the time-domain signal $s(t)$.

$$\mathbf{X}^{TF}[n,m] = \frac{1}{MN} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \mathbf{X}^{DD}[l,k] e^{j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)} \quad (4)$$

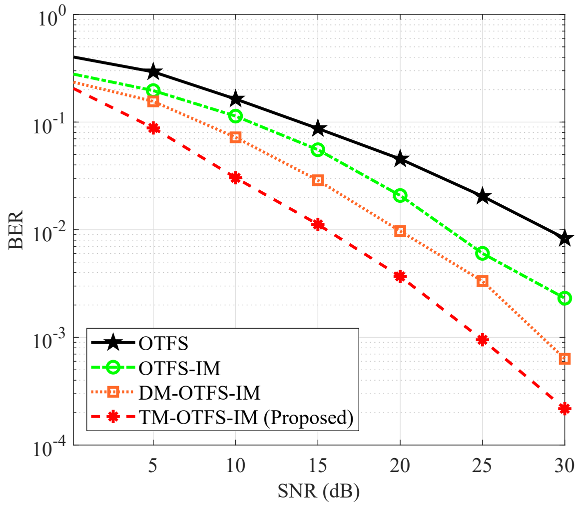

Fig. 3: BER comparison of the proposed TM-OTFS-IM with the DM-OTFS-IM, OTFS-IM, and OTFS

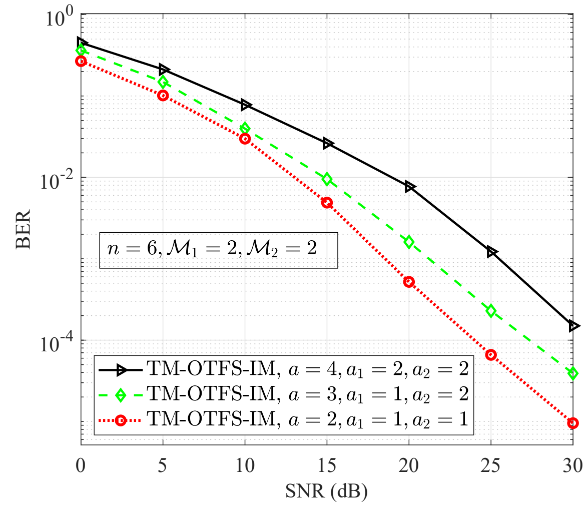

Fig. 4: BER performance of TM-OTFS-IM under different activation of DD resources.

TABLE II: Parameters for Simulation

Parameter	Value
Maximum Speed	500km/h
Number of multi-paths	5
Carrier frequency	2.45GHz
No.of Delay Resouces (M)	32
No.of Doppler Resouces (N)	16
Subcarrier spacing Δf	15KHz
Bandwidth	MΔf
Frame duration	N/Δf
Delay Profile	Extended Vehicular A (EVA)
Channel Ideal estimation	Ideal

where $n \in (0, 1, ..., N-1)$, $m \in (0, 1, ..., M-1)$.

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \mathbf{X}^{TF}[n, m] e^{j2\pi m \Delta f (t-nT)} g_{\text{tx}}(t - nT) \quad (5)$$

where $g_{\text{tx}}(t)$ is the transmit pulse function. The time domain signal will undergo doubly dispersive channel $h(\tau, \nu)$, given by.

$$h(\tau, \nu) = \sum_{p=0}^{P} h_p \delta(\tau - \tau_p) \delta(\nu - \nu_p) \quad (6)$$

where τ_p, ν_p and h_p, denotes the delay, Doppler shift and complex channel gain associated with p^{th} channel path, respectively. The time domain received signal is given by

$$r(t) = \iint h(\tau, \nu) e^{j2\pi\nu(t-\tau)} s(t - \tau) d\tau d\nu + z(t) \quad (7)$$

where $z(t)$ complex Gaussian noise. The received time-frequency (TF) domain signal $\mathbf{Y}^{TF}$ is derived from the time-domain signal through the application of the Wigner transform. Subsequently, this received TF domain signal is converted into a received Delay-Doppler (DD) domain signal $\mathbf{Y}^{DD}$ using the Symplectic Finite Fourier Transform (SFFT).

$$\mathbf{Y}^{TF}[n, m] = \int_{-\infty}^{\infty} g^*_{\text{rx}}(t - nT) r(t) e^{-j2\pi m \Delta f (t-nT)} dt \quad (8)$$

where $g^*_{\text{rx}}(t)$ is the received pulse function.

$$\mathbf{Y}^{DD}[l, k] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \mathbf{Y}^{TF}[n, m] e^{-j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)} \quad (9)$$

Now the signal $\mathbf{y} \in \mathbb{C}^{MN \times 1}$ is obtained by vectorizing the $\mathbf{Y}^{DD}$, which can be grouped as

$$\mathbf{y}^g = [y^g(1), \ldots, y^g(\zeta), \ldots, y^g(n)]^{\text{T}}, 1 \leq g \leq G. \quad (3)$$

After demodulation of DD domain symbols, a joint maximum likelihood (ML) detection algorithm is used to detect the information from the modulated symbols and indices of the modulated symbols.

$$\left(\hat{I}, \hat{\xi}\right) = \arg\min_{I, \xi} \sum_{\eta=1}^{n} |y(\eta) - h(\eta)x(\eta)|^2 \quad (10)$$

The predefined lookup table is shared and known by both the transmitter and receiver. An example of a lookup table for $a = 2, a_1 = 1, a_2 = 1$ and $n = 4$ is given in TABLE I.

III. Simulation Results

In Fig. 3, a BER comparison of the proposed TM-OTFS-IM with the DM-OTFS-IM, OTFS-IM, and OTFS is presented. A doubly spread channel model $h(\tau, \nu)$ is considered, with parameters given in TABLE II. The modulation order in each technique is selected to maintain a constant SE of 8 bits/sec/Hz to ensure a fair comparison of BER. Specifically for the proposed TM-OTFS-IM ($a = 2, a_1 = 1, a_2 = 1, \mathcal{M}_1 = 8, \mathcal{M}_2 = 2$), DM-OTFS-IM ($a_1 = 2, a_2 = 2, \mathcal{M}_1 = 4, \mathcal{M}_2 = 2$), OTFS-IM ($a = 3, \mathcal{M} = 4$), OTFS ($\mathcal{M} = 4$) with $n = 4$ total number of resources per group. As observed in Fig. 3, the proposed technique achieves gains of 5 dB, 8 dB, and 14 dB over Dual-Mode OTFS-IM, OTFS-IM, and OTFS, respectively, at a BER level of 10^{-2}. TM-OTFS-IM exhibits superior performance in terms of BER compared to the other techniques mentioned. This is because TM-OTFS-IM can transfer more bits through index information while utilizing fewer active resources. Fig. 2 shows the constellations symbols for the proposed technique, where one DD resource is modulated with $\xi(\mathcal{M}_1)$ and the other with modulation $\xi(\mathcal{M}_2)$. For all the results presented in this paper, the two constellations are chosen such that they have no common points and can be distinctly recognized by the receiver.

In Fig. 4, the BER performance of a proposed TM-OTFS-IM system is shown for different numbers of activation of DD resources. When the two DD resources are activated ($a = 2, a_1 = 1, a_2 = 1$), one with modulation symbols $\xi(\mathcal{M}_1)$ and the second with modulation symbols $\xi(\mathcal{M}_2)$, the gain of 3 dB and 6 dB in SNR is observed at a BER level of 10^{-4}, as compared to other activation of resources like $a = 3, a_1 = 1, a_2 = 2$ and $a = 4, a_1 = 2, a_2 = 2$ respectively. There is a trade-off between the number of active DD resources and BER reduction. As the active resources for transmission increase, the BER also increases. The total number of resources per group for simulation is $n = 6$ and modulation order of both constellation is $\mathcal{M}_1 = 2$ and $\mathcal{M}_2 = 2$.

IV. Conclusion

This study proposes a novel TM-OTFS-IM technique for improving BER and energy efficiency while keeping the spectral efficiency constant. Considering the BER values across different SNR levels, TM-OTFS-IM appears to offer superior performance compared to OTFS, OTFS-IM, and DM-OTFS-IM in terms of BER, making it a promising choice for communication systems where reliability and robustness are crucial.

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